A critical re-examination of the quantum Zeno paradox

1992 J. Phys. A: Math. Gen. 25 657

(http://iopscience.iop.org/0305-4470/25/3/022)

J. Phys. A: Math. Gen. 25 (1992) 657–664. Printed in the UK

A critical re-examination of the quantum Zeno paradox

D Home† and M A B Whitaker

Department of Pure and Applied Physics, The Queen's University of Belfast, Belfast BT7 1NN, UK

Received 8 July 1991

Abstract. The quantum Zeno *paradox* is defined as a negative-result experiment involving a macroscopic apparatus, and distinguished from other quantum Zeno processes. It is demonstrated that collapse of state-vector is not a requirement for the paradox, which is independent of interpretation of quantum theory. Gedanken experiments are outlined which illustrate the key features of the paradox, and its implications for the realist interpretation are discussed.

1. Introduction

In recent years there has been considerable discussion of the quantum Zeno effect (see, for example, Chiu *et al* 1977, Peres 1980, Joos 1984, Home and Whitaker 1986). The recent experiment of Itano *et al* (1990) has led to further analysis by Peres and Ron (1990), Petrovsky *et al* (1990), Ballentine (1990a, 1991) and Itano *et al* (1991).

There remain, though, aspects which we think require further clarification. First, the term 'quantum Zeno effect' (or 'paradox') has been used to describe processes of a very different nature, and this has led to considerable confusion. Secondly, the precise assumptions required for the effect to occur remain obscure, in particular the requirement or otherwise of a collapse of state-vector. Thirdly, the concept of 'continuous measurement' has remained obscure. We consider each of these questions in the following three sections of this paper, and in the fifth section discuss the implications of the quantum Zeno effect for a realist interpretation of the state-vector.

2. Classification of quantum Zeno processes

We feel it useful to introduce the following classification of the various examples of quantum Zeno effect that have been discussed in the literature. In the first type, survival probability of an evolving quantum system is predicted to be altered by a process of repeated observations with a macroscopic apparatus. Since it seems a remarkable prediction of quantum theory that the mere presence of an observing apparatus should affect the behaviour of a system, this type of process has been termed the quantum Zeno *paradox*, and we consider that the term should be reserved for this type of case.

In the second, the time evolution of a quantum system is affected by interaction with an external field or other agency, but no correlation with states of a macroscopic

† On leave from: Department of Physics, Bose Institute, Calcutta 700009, India.

0305-4470/92/030657+08$04.50

object is involved. In contrast to the first class, it is inherently reasonable that an interaction should have such an effect, and several such calculations and experiments have been performed. Greenland and Lane (1989) have recently reviewed the area thoroughly. Such work is often extremely interesting, but not difficult to understand in principle. We suggest that the term 'quantum Zeno *effect*' may be more suitable for this type of process.

It should be mentioned that other authors (Kraus 1981, Joos 1984, Sudbery 1986) have used the terms 'watched-pot' and 'watchdog' for these processes; there does not appear to be, in general, any one-to-one correspondence between their individual classifications, or between theirs and ours.

In discussion of the experiments of Itano *et al* (1990), several authors (Peres and Ron 1990, Petrovsky *et al* 1990, Ballentine 1991) were able to show that the inhibition of transitions demonstrated in the experiment could be explained using only the Schrödinger equation and without use of any macroscopic measuring device in the analysis. Our classification would therefore describe it as a particularly interesting example of the quantum Zeno effect, not, as originally suggested, of the quantum Zeno paradox. (It is emphasized that these experiments are certainly not, again as originally suggested, a proof of collapse of state-vector.) It should not be suggested, though, that these other arguments explain away, or refute, or make trivial, the original prediction of the paradox by Chiu *et al* (1977). The experiments of Itano *et al*, and the theory of Chiu *et al* are just about different types of situation.

Similarly, Dehmelt (1986a, b) has discussed an interesting experiment which he calls the continuous Stern–Gerlach effect. What he describes as a 'measurement' process on the electron is initiated by imposition of a minute magnetic field, and the process does not require any macroscopic device. When he predicts a decrease in transition rate as the measurement time is decreased, then he is demonstrating the quantum Zeno effect. His claim that this is 'the unspectacular resolution of "Zeno's paradox"' is, we suggest, misleading. Again his predictions do not relate to the paradox, in its original terms, and as we have defined it here.

3. Analysis of the quantum Zeno paradox

Most discussion of the quantum Zeno paradox has used the idea of collapse of state-vector at a measurement (e.g. Chiu *et al* 1977). (Throughout this paper, we use the term 'collapse' to mean that the combined state-vector of system and apparatus actually becomes a mixed state at the end of the measurement.) Most authors have concluded, in fact, that it is an essential component of any argument for the paradox.

Proponents of ensemble interpretations of quantum theory (e.g. Ballentine 1970) are extremely opposed to the very idea of collapse, since it lies outside the Schrödinger equation, and implies that quantum evolution is of two distinct types—usually by the Schrödinger equation, except at a measurement, when collapse must be invoked. They claim that ensemble interpretations do not require the idea of collapse, and may then suggest that this argument demolishes any proof of the quantum Zeno paradox (again assuming that such must rely on collapse) (Ballentine 1990b (p 237), 1991).

We show here that the quantum-mechanical prediction of the Zeno paradox is, in fact, in no way reliant on any assumption of collapse (Whitaker 1989). It follows simply from the correlation between states of the measured system and the measuring apparatus, and is independent of choice of interpretation of quantum theory.

For simplicity we restrict our analysis to the case where the decayed state is represented by a single energy level with state-vector $|\psi_1\rangle$, the initial state being $|\psi_0\rangle$, with $\langle\psi_0|\psi_1\rangle = 0$.

If $|\psi_0\rangle$ is allowed to evolve for a time T uninterrupted by any external intervention, the final state-vector will be given by

$$|\psi(T)\rangle = \exp(-\mathrm{i}HT)|\psi_1\rangle \tag{1}$$

and, in our model, this may be written as

$$|\psi(T)\rangle = \alpha(T)|\psi_0\rangle + \beta(T)|\psi_1\rangle. \tag{2}$$

Thus the survival probability, P_s, that is, the probability of finding the system in the original state $|\psi_0\rangle$, is given by

$$P_s = |\alpha(T)|^2 \tag{3}$$

and after the measurement to determine survival or otherwise, and without assuming collapse, the comined state-vector of system and apparatus is given by

$$|\Psi(T)\rangle = \alpha(T)|\psi_0\rangle|A_0\rangle + \beta(T)|\psi_1\rangle|A_1\rangle \tag{4}$$

where $|A_0\rangle$ and $|A_1\rangle$ are orthogonal apparatus states.

Now consider an intermediate measurement to determine survival or otherwise at $t = T/2$. After such a measurement

$$|\Psi(T/2)\rangle = \alpha(T/2)|\psi_0\rangle|A_0\rangle + \beta(T/2)|\psi_1\rangle|A_1\rangle. \tag{5}$$

Then the system is allowed to evolve free from measurement until $t = T$ when

$$|\Psi(T)\rangle = \alpha(T/2)\exp(-\mathrm{i}HT/2)|\psi_0\rangle|A_0\rangle + \beta(T/2)\exp(-\mathrm{i}HT/2)|\psi_1\rangle|A_1\rangle. \tag{6}$$

A measurement at $t = T$ will then show that survival probability at $t = T$ is given by

$$\begin{aligned} P'_s &= |\langle\psi_0|\langle A_0|\Psi(T)\rangle|^2 \\ &= |\langle\psi_0|\langle A_0|\alpha(T/2)\exp(-\mathrm{i}HT/2)|\psi_0\rangle|A_0\rangle \\ &\quad + \langle\psi_0|\langle A_0|\beta(T/2)\exp(-\mathrm{i}HT/2)|\psi_1\rangle|A_1\rangle|^2. \end{aligned} \tag{7}$$

Since $\langle A_0|A_1\rangle = 0$, the survival probability reduces to

$$\begin{aligned} P'_s &= |\alpha(T/2)|^2|\langle\psi_0|\{\alpha(T/2)|\psi_0\rangle + \beta(T/2)|\psi_1\rangle\}|^2 \\ &= |\alpha(T/2)|^4. \end{aligned} \tag{8}$$

We may generalize the analysis to the case of measurements at $T/n, 2T/n, \ldots, T$, where

$$P'_s = |\alpha(T/n)|^{2n}. \tag{9}$$

It may easily be seen that for an exponentially decaying system where $|\alpha(T)|^2$ is an exponentially decreasing function of time,

$$P_s = P'_s. \tag{10}$$

On the other hand, for the most usual case where $\langle H\rangle$ and $\langle H^2\rangle$ are finite, we may write

$$|\alpha(T/n)|^2 = 1 - (\Delta H)^2(T/n)^2 \ldots \tag{11}$$

where

$$(\Delta H)^2 = \langle H^2 \rangle - \langle H \rangle^2 \tag{12}$$

and hence

$$P_s' = [1 - (\Delta H)^2 (T/n)^2 \ldots]^n \tag{13}$$

and $P_s' \to 1$ as $n \to \infty$.

Indeed, for a general hypothetical power-law decay for small time

$$\alpha(T) = 1 - kT^m \tag{14}$$

it is easy to see that, if measurements are made at $T/n, 2T/n \ldots T$, the decay probability at T may be written, for small T/n, as

$$P_s' = 1 - kT^m n^{1-m}. \tag{15}$$

For $m > 1$, $P_s' \to 1$ as $n \to \infty$. For $m = 1$, P_s' is equal to P_s; this is reasonable, because the exponential, for which the equality holds in general, reduces to a t-dependence for small times. For $0 < m < 1$, though, decay is increased rather than decreased by the measurements. (For $m < 0$, (14) cannot apply.)

It should be noted that, in the above treatment, no collapse has been assumed. The time evolution has been considered according to the Schrödinger equation, and superposition has been maintained at all levels. The key input has been the correlation between states of system and of apparatus, and, in particular, the orthogonality of the macroscopically distinguishable states of the apparatus, $|A_0\rangle$ and $|A_1\rangle$.

To conclude this section we use an explicit formalism to demonstrate differences and similarities between the paradox and effect as defined in section 2, and also between different types of process both categorized as effects.

Following Peres (1980) we consider a two-level system. We write the state-vector of the decaying species as $a_0|\psi_0\rangle + a_1|\psi_1\rangle$, where a_0 and a_1 are appropriate coefficients for surviving and decayed states respectively. If V_{10} and V_{01} are matrix elements of the Hamiltonian of the system, then the Schrödinger equation will give

$$\dot{a}_0 = -(\mathrm{i}/\hbar) V_{01} a_1 \exp[\mathrm{i}(E_0 - E_1)t/\hbar] \tag{16}$$

$$\dot{a}_1 = -(\mathrm{i}/\hbar) V_{10} a_0 \exp[\mathrm{i}(E_1 - E_0)t/\hbar]. \tag{17}$$

It is (16) that is important from our point of view. It indicates that a_0 cannot change, that is, decay cannot take place, if the effect of the right-hand side is reduced to zero. In the genuine quantum Zeno paradox (as define in section 2), the effect is so reduced by correlating the coefficients with orthogonal macroscopic detector states.

When one turns to the quantum Zeno effect, there are two distinct cases. In the first, that studied by Peres (1980) and Greenland and Lane (1989), the right-hand side of (16) is rendered ineffective by disturbances, either inside the system, or from external collisions, which introduce rapid random changes of relative phase between $|\psi_0\rangle$ and $|\psi_1\rangle$. In the experiments of Itano *et al* (1990), a different mechanism is in operation, the correlation of atomic states with orthogonal states of the electromagnetic field (Peres and Ron 1990, Petrovsky *et al* 1990). Mathematically, of course, this has more in common with the quantum Zeno paradox than the first type of quantum Zeno effect. Physically and conceptually, however, it must be grouped with the effects, because it results from an easily understandable microscopic process, rather than from a conceptually problematic negative-result process involving a macroscopic apparatus.

From (16), it is easy to see that a collapse of state-vector will also cause $\dot{a}_0$ to be reduced, because at a collapse the right-hand side is actually put equal to zero. But, as shown immediately above, the collapse assumption is certainly not necessary.

4. Gedanken experiments

The considerations of section 3 have been formal. It certainly remains to be demonstrated that a mere 'observation' can really have the properties of a quantum-mechanical measurement (Peres 1986). Beyond that, the question of 'continuous measurement' has also been the subject of much discussion, attention being directed, for example, to the finite response time of any observing device.

We would emphasize that, for any observations to convey much meaning, they must give definite information on whether decay has taken place. Thus, while merely positioning a single detector in the vicinity of a decaying atom is of no use, one may imagine a sphere whose inner surface is covered with detectors, with the decaying atom at its centre. It will be arranged that any detection event from any point on the surface gives rise electronically to a (macroscopic and permanent) black mark on a recording strip. Then the presence of such a mark at time T indicates that the atom has decayed in the period up to $(T-r/v)$, where r is the radius of the sphere, and v is a value for the speed of the decay product, determined by its energy (assumed unique, for simplicity). The absence of a mark indicates that decay has not taken place in the same period. Thus one predicts a one-to-one correlation between occurrence of a quantum event and macroscopic registration, and this is exactly the definition of a quantum measurement.

The foregoing does not, however, shed much light on the question of continuous measurement. Before reporting a gedanken experiment which does so, we introduce the idea of the generalized Zeno paradox. The crucial element of the quantum Zeno paradox as usually defined is the reduction of decay, and, in particular, the possibility of its complete elimination. To study this, even in principle, we require practically continuous measurement, and the ability to work in a t^m-region of decay with $m>1$, as mentioned above.

Certainly it is recognized that such an experiment would be the most dramatic manifestation of the ideas involved. Nevertheless, if the principle under consideration is taken to be an influence on the decay rate of a system caused by presence of detection devices, this principle will be manifested in any such change. And as already stated, some change will be predicted for any decay other than a perfect exponential, and no quantum decay can ever be a perfect exponential. By this, we mean not just that there must be small-t and large-t regions where the decay is markedly different from exponential, but that, even in the so-called exponential regions, the exponential can never be exact. It will be freely admitted that the strenuous search for such divergences has not been successful (Greenland 1988, Norman *et al* 1988) but they certainly must exist in principle.

We claim, then, that to establish this generalized Zeno paradox all that is required is to check decay statistics for a decaying species with various sequences of observation times. The differing sequences should give different patterns of decay. Any need to establish, and work, inside a t^2-region of decay is thus removed.

A gedanken experiment which will study decay at discrete times will now be outlined. Again the detectors cover the inside surface of a sphere, but now the surface is made of a balloon-type of material, and its radius may be changed extremely rapidly, while maintaining its shape and the position of its centre.

If we wish to test decay or survival at T/n, $2T/n \ldots T$, the radius of the detector balloon is held at rather greater than Tv/n from $t=0$ to $t=T/n$, when it is reduced rapidly to a very small value, and then immediately increased to Tv/n again. No detection can be made while the radius is Tv/n, but for an atom to be found to have decayed in the period before time T/n, a corresponding detection will be made during the sweep-in of the balloon. Similar contractions and immediate expansions are made at time $2Tv/n$ and so on. Thus decay statistics may be built up.

The whole experiment can then be repeated for a different value of n, and the prediction of the generalized Zeno paradox is that the results should, in principle, differ.

It is easily seen that, for this gedanken experiment, the idea of continuous measurement is completely absent. The experiment to demonstrate the quantum Zeno paradox is seen to be just a negative-result experiment. Indeed, there is a strong analogy between this gedanken experiment, and a Stern–Gerlach experiment with a detector at the end of one beam only. In the Stern–Gerlach, one either does or does not detect a particle, and the awkard question is how a particle which passes along the undetected beam 'knows', in the absence of any interaction, that it is expected to assume the value of S_z appropriate to that beam. In the quantum Zeno experiment also, one either does or does not detect a particle at each sweep (from a given decaying atom). If an atomic system has been observed not to have decayed by time T/n, it has taken part in no interaction with the detector, yet 'knows' that it must now start its decay profile from $t=0$ again, with a new t^2-region in the normal case. Of course, as shown in section 3 for the Zeno case, and also in the Stern–Gerlach case, the formal explanation involves correlation of atomic states with orthogonal detector states, but this does not make the effect any less difficult to understand from the physical point of view.

5. A realist view of the quantum Zeno paradox

This brings us to the realist approach to this type of decay experiment. Any realistic description that is at all natural is bound to contradict quantum theory, and the reason why such experiments are difficult to understand, the reason why we are inclined to use the word 'paradox', is that it is very attractive to accept the realist position in this area. In this way, it is perhaps rather different from the case of a spin-1/2 particle where it may not seem too unnatural to believe the spin to be in a 'quantum' superposition, rather than a 'realist' mixed state. The decay case is more nearly analogous to that of Schrödinger's cat, where the realist view that the cat is not in a superposition of dead and live states seems much more natural.

For the case of decay, the quantum description is in terms of a superposition of clearly physically distinguishable 'surviving' and 'decayed' states. The latter state includes a decay particle or particles which may have travelled a macroscopic distance from the decaying species, and this fact makes the idea of the superposition particularly unappealing.

The alternative realist description must maintain that, after any time of decay t, the ensemble is heterogeneous, consisting of individual systems that have either decayed or survive. If one further insists that the probability of survival from t_1 to t_2 depends only on the condition of the system at t_1, and not on any previous history, the exponential decay law must result (Ballentine 1990b, p 234). Though such an assumption is plausible, it is certainly not obligatory for a realist position; individual decay parameters may, of course, be allocated to decaying atoms so as to mimic a t^2-region, or any other decay law one wishes, though such mimicry will probably appear artificial. Thus, even before any idea of measurement, the realist view of decay is on the one hand appealing, but on the other problematic.

Let us now consider the effect of the measurement process. To reproduce the predictions of quantum theory, one must imagine that the decay parameters of the surviving atoms are modified by the observing process. Yet there has been no local interaction affecting the decaying system, and, in particular, its dynamical attributes such as energy and momentum remain unaffected.

There would thus seem to be an incompatibility between quantum theory and local realism in this case, unless one considers the wavefunction to be a physically real field, and suggests some causal mechanism for this field to give rise to the Zeno process. In terms of the quantum potential approach (Bohm and Hiley 1987), one would argue that modification of the total coherent wavefunction by coupling with the apparatus states leads to a changed quantum potential, and hence the decay rate of an individual decaying system may become affected. However, this is merely a formal way of discussing the problem, and, to constitute a satisfactory realist explanation of the Zeno paradox, it would need to be supplemented by a more detailed physical explanation in terms of a causal spacetime description based on a clear understanding of the physical origin of the quantum potential.

6. Conclusions

Much of our analysis has been fairly abstract, and our proposed experiments of the gedanken variety. The most interesting question remaining is—will the study of the quantum Zeno paradox always remain a question of principle devoid of experimental test? (We are here using our definition of section 2, and, of course, contrasting the paradox with the quantum Zeno effect as also defined there; the latter has already been subject to several experimental tests.)

The one important step to making the quantum Zeno paradox accessible experimentally would be the discovery or invention of systems whose decay is far from exponential. Greenland (1988) has demonstrated the very great difficulty likely to be found in achieving such a decay. Yet he is not totally devoid of hope, finding promise in such areas as the near-threshold photodetachment of electrons from negative ions using highly stabilized lasers. In another interesting paper, Sluis and Gislason (1991) have studied decay from a system with energy distribution given by a truncated Lorentzian. They do not predict a quantum Zeno paradox (in the sense of total inhibition of transitions), but do suggest that rapid measurements might reduce the decay rate by a factor of 2.

We hope we have demonstrated that the quantum Zeno paradox is of very considerable theoretical interest in the analysis of quantum theory, as interesting, in its own way, as the famous Einstein–Podolsky–Rosen (EPR) 'paradox'. We have shown that the prediction of such an effect is a direct result of orthodox quantum theory, and makes no use of such questionable concepts as collapse of state-vector or continuous measurement. It is thus entirely independent of interpretation of quantum theory.

It is extremely desirable that the feasibility of carrying out experiments to test these predictions should be seriously considered. If the results did confirm the predictions, this would present considerable problems for realism. However, if they did not do so, that would constitute a serious problem for quantum theory in the way in which it handles measuremental results.

Acknowledgment

One of the authors (DH) gratefully acknowledges the Fellowship of the Commission of the European Communities, and the support provided by the Department of Science and Technology (Government of India) for his research work.

References

Ballentine L E 1970 *Rev. Mod. Phys.* **42** 358
—— 1990a *Found. Phys.* **20** 1329
—— 1990b *Quantum Mechanics* (Englewood Cliffs, NJ: Prentice-Hall)
—— 1991 *Phys. Rev.* A **43** 5165
Bohm D and Hiley B J 1987 *Phys. Rep.* **144** 323
Chiu C B, Sudarshan E C G and Misra B 1977 *Phys. Rev.* D **16** 520
Dehmelt H 1986a *Proc. Natl Acad. Sci. USA* **83** 2291
—— 1986b *Proc. Natl Acad. Sci. USA* **83** 3074
Greenland P T 1988 *Nature* **335** 298
Greenland P T and Lane A M 1989 *Phys. Lett.* **134A** 429
Home D and Whitaker M A B 1986 *J. Phys. A: Math. Gen.* **19** 1847
Itano W M, Heinzen D J, Bollinger J J and Wineland D J 1990 *Phys. Rev.* A **41** 2295
—— 1991 *Phys. Rev.* A **43** 5168
Joos E 1984 *Phys. Rev.* D **29** 1626
Kraus K 1981 *Found. Phys.* **11** 547
Norman E B, Gazes S B, Crane S G and Bennett D A 1988 *Phys. Rev. Lett.* **60** 2246
Peres A 1980 *Am. J. Phys.* **48** 931
—— 1986 *Information Complexity and Control in Quantum Physics* ed A Blaquiere, S Diner and G Lochak (Udine: CISM)
Peres A and Ron A 1990 *Phys. Rev.* A **42** 5720
Petrovsky T, Tasaki S and Prigogine I 1990 *Phys. Lett.* **151A** 109
Sluis K M and Gislason E A 1991 *Phys. Rev.* A **43** 4581
Sudbery A 1986 *Quantum Concepts in Space and Time* ed R Penrose and C J Isham (Oxford: Clarendon) p 65
Whitaker M A B 1989 *Eur. J. Phys.* **10** 71